Field of View Calculations for SFXC

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1 Introduction

The advent of the EVN software correlator at JIVE (SFXC) has greatly expanded wide-field mapping capabilities within the EVN, by removing hard limits to the maximum number of frequency points per subband and to the minimum integration time inherent in the EVN MkIV data processor. The principal limitations to wide-field mapping are the smearing effects due to averaging the data in time and/or frequency during the correlation itself (or at some subsequent point). Because there is a finite integration time and a finite frequency-point width, each visibility in your FITS files actually corresponds to an area on the u-v plane: "radial" extent corresponding to frequency averaging and "azimuthal" extent to time averaging. Because increasing residual delays and rates ensue for Structures in the source plane at farther distances from the correlation phase center contribute increasing residual delays (*i.e.,* phase slope *vs* frequency) and rates (*i.e.,* phase slope *vs* time), coherent averaging over such finite-BW frequency points and finite-time integrations will lead to de-correlations; and hence to distortions in the resulting image. The EVN calculator (`www.evlbi.org/cgi-bin/EVNcalc`) now incorporates both field-of-view computations consistent with the various equations in this paper. Hopefully, you will find this description and these tools help take the mystery out of selecting such parameters when it's time to propose/correlate your experiment.

Here, we'll begin by reviewing the formulae for computing the fields of view resulting from bandwidth and time smearing (§2). Next, we place these purely mathematical results in context for SFXC (§3). With these two aspects in hand, we then tabulate the fields of view resulting from various configurations of observation/correlation parameters (§4).

2 Field of View Formulae

The effects of time- and frequency-averaging during correlation (or later) act to smear out irretrievably the response of structure located away from the phase center of the correlation. The distance at which this becomes significant is a function of the correlation parameters, and of course your tolerance for such distortions. For this review, we'll take the formulae from Wrobel (1995), §21.7.5. These provide the maximum field of view having no more than a 10% decrease in the response to a point source. We recast these formulae in terms of correlation parameters that you can select. For more information about the derivation and behavior of time- and frequency- (also called bandwidth-) smearing, see Bridle & Schwab (1989). Further, image distortion due to the non-coplanarity of the observing array, which we don't dwell on here, is discussed in Perley (1989).

2.1 FoV in terms of the synthesized beam, θ

$$\begin{aligned}
\text{Bandwidth}: \quad FoV &\lesssim 0.8\,\theta\,\frac{\nu_0 N_\nu}{BW_{\rm SB}} \quad \text{or} \quad \lesssim 0.8\,\theta\,\frac{\nu_0 N_{\rm SB} N_\nu}{BW_{\rm tot}} \\
\text{Time}: \quad FoV &\lesssim \frac{9000}{t_{\rm int}}\,\theta \qquad (1)
\end{aligned}$$

Here, N_ν is the number of frequency points per subband and $N_{\rm SB}$ is the number of subbands. A subband can be thought of an IF in `aips` usage: different upper- or lower-sidebands from each BBC count as subbands, but the number of polarizations does not enter the picture. $BW_{\rm SB}$ is the subband bandwidth. $BW_{\rm tot}$ is the total sampled bandwidth ($= N_{\rm SB} \cdot BW_{\rm SB}$), ν_0 is the sky frequency (pick lower edge of lowest SB for most conservative estimate), and $t_{\rm int}$ is the integration time. Other parameters have mks units.

Aside 1

Note that we take the convention here that the number of polarizations does not enter into the definition of $BW_{\rm tot}$; the total recording rate (in Mb/s) for b-bit Nyquist sampling would be $2\,bN_{\rm SB}BW_{\rm SB}N_{\rm pol_{\|}}$. Here, $N_{\rm pol_{\|}}$ is the number of observed polarizations; the "‖i" subscript intends to draw a distinction between the observed and correlated polarizations. For a dual-polarization observation, the former would be 2, but the latter could be 2 or 4, depending on whether the cross-polarization products were desired in the correlated data. Hence $N_{\rm pol_{\|}}$ only counts the parallel-hand polarizations, removing any such ambiguity in §3.

The parameters in eq.(1) that can be controlled during correlation are N_ν, $t_{\rm int}$, and $N_{\rm SB}$. The latter will be set in most cases by how you scheduled your experiment, but the first two are entirely independent of the observations.

2.2 FoV in terms of arc-seconds

Take $\theta \simeq \lambda/B$, where λ is the wavelength associated with ν_0 and B is the longest baseline in the array. We then obtain, after converting from radians to arc-seconds, with B in units of 1000 km, λ in cm, $t_{\rm int}$ in seconds, and all BW in MHz:

$$\begin{aligned} \text{Bandwidth}: \quad FoV &\lesssim 49.''5\,\frac{1}{B}\,\frac{N_\nu}{BW_{\rm SB}} \quad \text{or} \quad \lesssim 49.''5\,\frac{1}{B}\,\frac{N_{\rm SB}N_\nu}{BW_{\rm tot}} \\ \text{Time}: \quad FoV &\lesssim 18.''56\,\frac{\lambda}{B}\,\frac{1}{t_{\rm int}} \end{aligned} \tag{2}$$

Note that the bandwidth smearing is now independent of the observing frequency when expressing the field of view in arc-seconds. The scalings to keep in mind for the field-of-view within which distortion is limited to a given level are:

- BW- larger FoV with increasing N_ν
- time- larger FoV with decreasing $t_{\rm int}$, increasing λ
- Both: smaller FoV with increasing baseline length

2.3 FoV as a fraction of a single-dish beam

A single-dish beam will be $\simeq \lambda/D$. Instead of evaluating the fields of view resulting from the various types of smearing in arc-seconds as in the previous subsection, we can also form the ratios of the field of view to a single-dish beam. With B in units of 1000 km, D in m, λ in cm, $t_{\rm int}$ in seconds, and all BW in MHz, we obtain:

$$\begin{aligned} \text{Bandwidth}: \quad \text{Frac. s.d. beam} &\lesssim 0.024\,\frac{D}{B}\,\frac{N_\nu}{BW_{\rm SB}\lambda} \quad \text{or} \quad \lesssim 0.024\,\frac{D}{B}\,\frac{N_{\rm SB}N_\nu}{BW_{\rm tot}\lambda} \\ \text{Time}: \quad \text{Frac. s.d. beam} &\lesssim 0.009\,\frac{D}{B}\,\frac{1}{t_{\rm int}} \end{aligned} \tag{3}$$

3 Correlator Capabilities

3.1 Capacity

SFXC has greatly expanded available parameter space for correlation, and in turn has greatly simplified the approach you can take to planning the observation set-up and correlation parameters for your experiments. In a nutshell, there are not longer any explicit trade-offs to be made among $N_{\rm sta}$, $N_{\rm SB}$, N_ν, $N_{\rm pol}$, and $t_{\rm int}$ to ensure that the data will “fit” into the correlator.

There remains one fundamental limitation for $t_{\rm int}$: it cannot be smaller than the time associated with an FFT in order to provide the desired N_ν:

$$t_{\rm int} \geq N_\nu / BW_{\rm SB} \tag{4}$$

It is expected that this condition would pose a practical obstacle only in exceptional cases (*e.g.,* the only time this has been operationally relevant to date was in a ms-pulsar observation, in which multiple bins were desired across the pulse itself).

3.2 Output

Of course, one disadvantage of short $t_{\rm int}$ and/or large N_ν is that the size of your output data can grow quickly. A rule of thumb for the anticipated size of your FITS files per hour of observing is:

$$\simeq \frac{N_{\rm sta}(N_{\rm sta}+1)N_{\rm SB}N_\nu N_{\rm pol}\cdot f}{74565.4\cdot t_{\rm int}} \text{ GB per hour observing.} \tag{5}$$

where f is a scaling factor that empirically has been seen to be ~ 1.4 for $N_\nu \geq 1024$ and 1 otherwise. This size includes both the baselines and auto-correlations.

Note that if you'll be processing your correlated data through `aips`, there is a separate limit in the standard distribution of $N_{\rm SB}N_\nu N_{\rm pol} \leq 132096$ in the include file `PUVD.INC`.

4 Examples

Table 1 below shows the fields of view in arcseconds resulting from various combinations of the parameters in eqs.(2) & (3). Remember, all these numbers are ultimately based on the formulae from Wrobel (1995), so all correspond to a field of view that has no more than a 10% decrease in the response to a point source. If your needs are different, you'll have to adjust the tabulated fields of view accordingly (*e.g.,* see Fig.13–1 in Bridle & Schwab (1989) for a graph of the behavior of the peak response loss due to bandwidth smearing). Here, we consider:

$BW_{\rm SB}$ = 32 MHz, 16 MHz, 2 MHz

N_ν = 2048, 512, 32

B = 2500 km, 10000 km — Western European baselines; global baselines

$t_{\rm int}$ = 1 s, 0.25 s

We can see from table 1 that the time smearing is usually the limiting factor more often than is bandwidth smearing. The ratio of the two field of views can be easily computed from eq.(2), with λ in cm, $BW_{\rm SB}$ in MHz, and $t_{\rm int}$ in seconds:

$$\frac{\text{Bandwidth } FoV}{\text{Time } FoV} \simeq 2.67\frac{N_\nu t_{\rm int}}{BW_{\rm SB}\lambda}. \tag{6}$$

Bandwidth smearing

$BW_{\rm SB}$	N_ν	$B = 2\,500$ km	$B = 10\,000$ km
32 MHz	2048	$1267\farcs20$	$316\farcs80$
32 MHz	512	$316\farcs80$	$79\farcs20$
32 MHz	32	$19\farcs80$	$4\farcs95$
16 MHz	2048	$2534\farcs40$	$633\farcs60$
16 MHz	512	$633\farcs60$	$158\farcs40$
16 MHz	32	$39\farcs60$	$9\farcs90$
2 MHz	2048	$20275\farcs20$	$5068\farcs80$
2 MHz	512	$5068\farcs80$	$1267\farcs20$
2 MHz	32	$316\farcs80$	$79\farcs20$

Time smearing

λ	$t_{\rm int}$	$B = 2\,500$ km	$B = 10\,000$ km
18.0 cm	1.00 s	$133\farcs20$	$33\farcs30$
18.0 cm	0.25 s	$532\farcs80$	$133\farcs20$
6.0 cm	1.00 s	$44\farcs40$	$11\farcs10$
6.0 cm	0.25 s	$177\farcs60$	$44\farcs40$
1.3 cm	1.00 s	$9\farcs62$	$2\farcs40$
1.3 cm	0.25 s	$38\farcs48$	$9\farcs62$

Table 1: Fields of View in arcseconds for some representative configurations of observation/correlation parameters.

References

[1] Bridle, A.H. & Schwab, F.R. 1989, "Wide Field Imaging I: Bandwidth and Time-Average Smearing," in *Synthesis Imaging in Radio Astronomy*, p.247.

[2] Perley, R.A. 1989, "Wide Field Imaging II: Imaging with Non-Coplanar Arrays," in *Synthesis Imaging in Radio Astronomy*, p.259.

[3] Wrobel, J.M. 1995, "VLBI Observing Strategies," in *VLBI and the VLBA*, p.411.